**Optimization Method - Assignment 3**

Q1.

1. For the Hessian matrix , . When , . So when , it is still a nonpositive definite matrix. When , .
2. If , for , it should satisfy .

So but not .

Q2.

We know that .

Then .

Then we should verify that satisfies the quasi-Newton equation which is: . Since:

Here which means satisfies the quasi-Newton equation.

Q3.

We know that in the BFGS for, , . Thus:

We know that , thus:

And we know that as all matrices are the positive definite. So the must be negative.

According to the Wolfe condition in this question:

And if , then:

If , then

To conclude, must be larger than 0.

Q4.

1. The Lagrange function of this problem is:

Thus:

When , , so this point is not a stationary point.

When , . In this case and cannot satisfy at the same time, so this point is not a stationary point.

When , . In this case if , then . So this point is not a stationary point.

1. From question (a), we know that the stationary point is . Then the Hessian matrix of the Lagrange funct**i**on is:

However, which means that it is not a positive definite matrix. So we cannot make conclusion immediately.

Since the constraint is active, we consider .

So we cannot make sure that the result of which means that is a saddle point.

Q5.

The question is:

And we should change the constrain to the form in :

Then the Lagrange function is:

We have:

And the KKT conditions for this problem is:

Then we should perform classification discussion:  
Case1: is active and others are not active. Then we have:

Then we can get:

Case2: is active and others are not active. Then we have:

Then we can get:

should be larger than 0, so this case is rejected.

Case3: is active and others are not active. Then we have:

Then we can get:

So is a KKT point.

Q6.

We know that are the convex function and   satisfies the first order necessary optimality conditions. Thus, we have:

Hence are convex, for other points:

Thus,

And we know that :

Therefore,   is the global minimizer.

Q7.

We know that the problem is:

And we apply the inverse barrier function:

We have:

Then we can get:

When :

Thus, the minimizer is

Q8.

We know that the problem is

Then we apply the penalty method:

We have:

So we can get:

When and ;

When and ;

When .99 and ;

When .999 and ;

Then using the augmented Lagrangian method:

We have:

Then the updated multiplier is:

So the has a limit and the .